ACMS Problem Solving Seminar - Fall 2006
Calculus: Problem Set II - Products and Integration
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Note:

9. Evaluate the infinite product, $3^{\frac{1}{3}} \cdot 9^{\frac{1}{9}} \cdot 27^{\frac{1}{27}}$.

10. The vertices of an n-gon are labeled by real numbers $x_1, \ldots x_n$. Let a, b, c, d be four consecutive labels. If $(a-d)(b-c) < 0$, then we may switch b with c. Decide whether this switching operation can be performed infinitely often.

11. Let $f(x)$ be a polynomial of degree n with real coefficients and such that $f(x) \geq 0$ for every real number x. Show that $f(x) + f'(x) + f''(x) + \ldots + f^{(n)}(x) \geq 0$ for every real number x.

12. Show that $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots + \frac{1}{\sqrt{n}} < 2\sqrt{n}$.

13. In presentation, we determined the value of $\prod_{k=2}^{\infty} \left(1 - \frac{1}{k^2}\right)$; solve the same problem by using a logarithmic transformation.

14. Determine $\prod_{k=2}^{\infty} \frac{k^3-1}{k^3+1}$.

15. Determine $\prod_{k=0}^{\infty} (1 + x^{2^k})$.

16. The *Reimann zeta function* is defined by

$$\zeta(z) = \sum_{k=1}^{\infty} \frac{1}{k^z}$$

for all complex numbers with real part greater than 1. Show that for real numbers $s > 1$,

$$\zeta(s) = s \int_1^{\infty} \frac{\lfloor x \rfloor}{x^{s+1}} dx$$

where $\lfloor x \rfloor$ denotes the *greatest integer function.*