

# ACMS Problem Solving Seminar - Fall 2005

## Problem Set 6 - Combinatorial Games

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**Note:** The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.<sup>1</sup>

37. The numbers 1 through 20 are written in a row. Two players take turns putting plus signs and minus signs between the numbers. When all such signs have been placed, the expression is evaluated. The first player wins if the sum is even, and the second player wins if the sum is odd. Who will win and how?
38. In super mario, there are 100 turtles randomly positioned on a platform (500 pixels long). When two turtles walk into one another, they walk along until the midpoint of the length of their bodies meet and then they reflect away from one another. If our turtles travel at a constant velocity of 500 pixels per minute, what is the longest possible time it could take for all turtles to fall off?
39. If  $a, b, c$  are the side lengths of a triangle such that

$$a^2 + b^2 + c^2 = ab + bc + ca$$

show that the triangle must be equilateral.

40. Two players take turns placing bishops on the squares of a chessboard, so they cannot capture each other (the bishops may be placed on squares of any colour). The player who cannot move loses. Who wins, and how?
41. Two people play the following game: they take turns placing quarters down on a rectangular table (without stacking or moving an already placed quarter). The player who cannot put down a quarter loses the game. Who will win?
42. The number 60 is written on a blackboard. We first list all divisors of 60, then player one selects a divisor, subtracts it from 60, erases 60 and writes the result in its place on the board. We list all divisors of the result, and player two proceeds as described above. The player who writes 0 loses. Who wins?
43. Does there exist a winning strategy for either player when “Chomp” is played on an  $m \times n$  bar of chocolate? (Hint: look for a pure existence proof)

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<sup>1</sup>Email GAD10@albion.edu for (non-spoiler) hints!