

ACMS Problem Solving Seminar - Fall 2005

Problem Set 5 - Extremal Proofs

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Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.¹

32. Every road in Mathsland is one-way. Every pair of cities is connected by exactly one direct road. Show that there is a city which can be reached from every other city, either directly or via at most one intermediate city.
33. It is known that there is one lighter counterfeit coin among a collection of n similar coins. What is the least number of weighings required to locate the coin?
34. A rectangular floor is covered by tiles of two types: there are tiles of size 2×2 and tiles of size 1×4 . One tile got smashed recently, and we have another tile of the opposite type available. Prove that we will not be able to move the tiles around to successfully re-tile the floor with the spare tile as a replacement.
35. Seven dwarfs are sitting around a circular table. There is a cup in front of each. There is milk in some of the cups, but all together we have 3 liters. One of the dwarves shares his milk uniformly with other cups. Proceeding counter-clockwise, each of the other dwarfs, in turn, does the same. After the seventh dwarf has shared his milk, the initial content of each cup is restored. Find the initial amount of milk in each cup.
36. Start with several piles of chips. Two players will move alternately. A move consists of splitting every pile with more than one chip in it into two piles. The one who cannot split any pile is the loser. Who wins if we have 6 piles, of size 4, 5, 6, 6, 3, and 8? In general?

¹Email GAD10@albion.edu for (non-spoiler) hints!