ACMS Problem Solving Seminar - Fall 2005
Problem Set 4 - Invariants

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Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.[1]

24. Two students are fighting over a chocolate bar during the weekly department colloquium, and Dr. Bollman suggests a solution. The chocolate bar has m by n squares of chocolate. The students will take turns breaking the bar along the divisions and they continue breaking until only the unit squares remain. The student who cannot make a break forfiets the chocolate. Is this game fair?

25. Who wins when "Chomp" is played on a $2 \times \infty$ chocolate bar? Assume both players are capable of swallowing an infinite amount of chocolate.

26. Let T be an equilateral triangle, and for any point p inside or on T, let d_1, d_2, and d_3 be the perpendicular distances from p to the three sides of T

    (a) Show that $d_1 + d_2 + d_3$ is independent of the chosen point p.

    (b) Is there any other acute triangle such that the value of $d_1 + d_2 + d_3$ is independent of the chosen point p?

27. There is a chip on each dot in the figure shown. In one move, you may simultaneously move any two chips by one place in opposite directions. The goal is to get all chips into one dot. When can this goal be reached?

28. In an 8×8 table one of the boxes is colored black and all others are white. Prove that one cannot make all the boxes white by recoloring the rows and columns. "Recoloring" is changing the color of all the boxes in a row or a column.

29. A biologist collects amoebae of three different types (A,B, and C, say) in a test tube. Two amoebae of any two different types can merge into one amoeba of the third type. After such merges only one amoeba remains in the test tube. What is its type, if initially there were 20 of type A, 21 of type B, and 22 of type C?

[1] Email GAD10@albion.edu for (non-spoiler) hints!

30. There is one stone at each vertex of a square. We are allowed to take away any number of stones from one vertex and add twice as many stones to the pile at one of the adjacent vertices. Is it possible to get 1989,1988,1990, and 1989 stones at consecutive vertices after a finite number of moves?

31. Three integers a, b, c are written on a blackboard. Then one of the integers is erased and replaced by the sum of the other two diminished by 1. This operation is repeated many times with the final result being 17, 1967, 1983. Could the initial numbers have been 2,2,2? Could they have been 3,3,3?