ACMS Problem Solving Seminar - Fall 2005
Problem Set 3 - Pigeonhole Principle

Giovanni DiMatteo

Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.[1]

16. A bridge club has ten members. Every day, four members of the club get together to play one game of bridge. Prove that after two years, there is some particular set of four members that has played at least four games of bridge together.

17. For what values of a does the system of equations

$$x^2 = y^2, \quad (x-a)^2 + y^2 = 1$$

have exactly zero, one, two, three, and four solutions, respectively?

18. If the game "Chomp" is played on an $n \times n$ chocolate bar, is there a sure winning strategy for either player? If so, find it and prove that it works.

19. Let A be any set of twenty distinct integers chosen from the arithmetic progression $1, 4, 7, \ldots, 100$. Prove that there must be two distinct integers in A whose sum is 104.

20. Given a set of $n+1$ integers between 1 and $2n$ (inclusive), show that two of them have no divisor in common.

21. Nineteen darts are thrown onto a dartboard which has the shape of a regular hexagon with side length one foot. Show that two darts are within $\sqrt{3}/3$ feet of eachother.

22. Let x be any real number. Prove that among the numbers

$$x, 2x, \ldots, (n-1)x$$

there is one that differs from an integer by at most $1/n$.

23. Prove that at a party of six people, there is a group of three mutual friends or a group of three mutual strangers.

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[1] Email GAD10@albion.edu for (non-spoiler) hints!