

ACMS Problem Solving Seminar - Fall 2005

Problem Set 2 - Parity

Giovanni DiMatteo

Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.¹

8. Is it possible to move a knight on a 5×5 chessboard so that it returns to its original position after having visited each square exactly once?
9. Show that for all positive real values of p , q , r , and s , that

$$\frac{(p^2 + p + 1)(q^2 + q + 1)(r^2 + r + 1)(s^2 + s + 1)}{pqrs}$$

cannot be less than 81.

10. Considering the diagram on the next page, is it possible to trace a path along the lines which traverses each line once and only once?
11. Is it possible to cover an 8×8 chessboard with 3×1 straight dominoes if we remove the upper right corner square?
12. A snail crawls along a plane with constant velocity, turning through a right angle every 15 minutes. Show that the snail can return to its starting position only after a whole number of hours.
13. Prove that $\sqrt[k]{n}$ is either an integer or irrational.
14. Let thirteen points P_1, \dots, P_{13} be given in the plane, and suppose they are connected by the segments $P_1P_2, P_2P_3, \dots, P_{12}P_{13}, P_{13}P_1$. Is it possible to draw a straight line which passes through the interior of these segments?
15. Suppose we have a 3×6 square array with one coin placed in each square. The coins may be moved one step horizontally or vertically, but you must move exactly two coins (not necessarily in the same direction or from the same square) simultaneously. Is it possible to stack all coins on one square, using these moves?

¹Email GAD10@albion.edu for (non-spoiler) hints!