ACMS Problem Solving Seminar - Fall 2005
Problem Set 1 - Introductory Problems
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Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.[1]

1. Prove that for all positive real numbers r and s,

$$\frac{r}{s} + \frac{s}{r} \geq 2$$

2. We are given 80 coins of the same denomination; we know that one of them is counterfeit and that it is lighter than the others. Locate the counterfeit using only four weighings on a pan balance.

3. The game "Chomp" is played in the following way: a rectangular chocolate bar consisting of $m \times n$ has a rotten cherry in the square in the lower left corner, and two players take turns picking a square and eating all chocolate above and to the right of that square (the loser is, of course, the one who is forced to eat the rotten cherry). If we play on a $2 \times n$ bar of chocolate, is there a winning strategy for either player? If so, describe it and, of course, prove that it works.

4. Prove that $\frac{x+y}{2} \geq \sqrt{xy}$ for all positive real numbers x and y. This is a special case of the Arithmetic-Geometric means inequality.

6. Prove that if five points are in or on a square with sides of length 1, then at least two points are no farther than $\frac{\sqrt{2}}{2}$.

7. Prove that $\tan(1°)$ is irrational.

8. A right circular cone has base of radius 1 and height 3. A cube is inscribed in the cone so that one face of the cube is contained in the base of the cone. What is the side-length of the cube?

[1] Email GAD10@albion.edu for (non-spoiler) hints!