

ACMS Problem Solving Seminar - Fall 2005

Presentation Material 4 - Invariants

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Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.

1. Can the board given in the diagram be covered by non-overlapping 1×4 dominoes?
2. Each of the numbers a_1, \dots, a_n is 1 or -1 , and we have

$$S = a_1a_2a_3a_4 + a_2a_3a_4a_5 + \dots + a_{n-1}a_na_1a_2 + a_na_1a_2a_3 = 0$$

Prove that $4|n$.

3. In the Parliament of Sikinia, each member has at most three enemies. Prove that the house can be separated into two houses, so that each member has at most one enemy in its own house.