

ACMS Problem Solving Seminar - Fall 2005

Presentation Material 2 - Parity

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Note: The following problems are taken from various sources, which are listed in pdf form on the ACMS problem solving seminar webpage.

1. Show that the diophantine equation

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d} + \frac{1}{e} + \frac{1}{f} = 1$$

has no solutions in odd integers a, b, c, d, e, f .

2. Prove that $\sqrt{2}$ is irrational.
3. Is it possible to tile a 10×21 board with 6×1 dominoes?
4. Let n be an odd positive integer: we write the numbers $1, 2, \dots, 2n$ on the board, and we are allowed to pick any two of these numbers (a and b say), erase them, and replace them with the positive integer $|a - b|$. Prove that after a finite number of moves, we will end up with a single number on the board and it will be odd.