Numerical Integration

Here are some numerical approximations to

$$\int_1^4 \sqrt{x}\,dx = \frac{2}{3}x^{3/2}\Big|_1^4 = \frac{2}{3}\cdot 8 - \frac{2}{3}\cdot 1 = \frac{14}{3} = 4.666666\cdots.$$

In the table below, n denotes the number of subintervals, $L(n)$ is the left-endpoint approximation, $R(n)$ is the right-endpoint approximation, $M(n)$ is the midpoint approximation, $T(n)$ is the trapezoid rule approximation, and $S(n)$ is the Simpson's rule approximation. Each approximation is accompanied by the difference between the approximation and the true value of the integral.

n	$L(n)$	error	$R(n)$	error	$M(n)$	error	$T(n)$	error	$S(n)$	error
2	3.871708	−.794959	5.371708	.705041	4.6884769	.0218103	4.6217082	−.0449584	4.6622777	−.0043890
4	4.280093	−.386574	5.030093	.363426	4.6724008	.0057341	4.6550926	−.0115741	4.6662207	−.0004460
8	4.476247	−.190420	4.851247	.184580	4.6681230	.0014564	4.6637467	−.0029200	4.6666314	−.0000353
16	4.572185	−.094482	4.759685	.093018	4.6670323	.0003657	4.6659349	−.0007318	4.6666643	−.0000024
32	4.619609	−.047058	4.713359	.046692	4.6667582	.0000915	4.6664836	−.0001831	4.6666665	$-1.5\cdot 10^{-7}$
64	4.643183	−.023484	4.690058	.023391	4.6666896	.0000229	4.6666209	−.0000458	4.6666667	$-9.7\cdot 10^{-9}$
128	4.654936	−.011731	4.678374	.011707	4.6666739	.0000057	4.6666552	−.0000114	4.6666667	$-6.1\cdot 10^{-10}$
256	4.660844	−.005862	4.672523	.005857	4.6666681	.0000014	4.6666638	−.0000029	4.6666667	$-3.8\cdot 10^{-11}$

1. What do you notice about the signs of the errors in the first four methods? Explain your observations in terms of features of the graph of the function $y = \sqrt{x}$.

2. What do you notice about the magnitudes of the errors for $L(n)$ and $R(n)$? How could $L(n)$ and $R(n)$ be combined to give a more accurate approximation? Where do you find this approximation in the table?

3. What do you notice about the magnitudes of the errors for $M(n)$ and $T(n)$? How could $M(n)$ and $T(n)$ be combined to give a more accurate approximation? Where do you find this approximation in the table?

4. For each of the methods, what is the ratio of the errors obtained when we double the number of subintervals? For each method there is a rule of thumb that the error is roughly proportional to the length of the subinterval raised to a certain power. What powers do you think pertain to the different methods?